**Cementing under High Temperature and High Pressure (HTHP) Conditions: a Fully Coupled Porothermoelastic Solution Using Plug and Abandonment (P&A) as Case Studies**

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**Abstract**

Using the primary cementing in P&A as example, the fully coupled porothermoelastic-osmosis-filtration (PTEOF) model is presented here. It has been demonstrated that under high-temperature and high pressure (HTHP) conditions, the induced pore pressure in the low permeability material such as cement can be very large to greatly reduce the effective stress and can even extend it to the tensile region. This reduced effective stress phenomenon will in general increase the probability of cement failure that may occur in different types and these phenomena are firstly described as permeability penalty. By the pairwise bivariate analysis, the properties that are important for the HTHP cementing are parsed out. Based on these results, two new concepts of permeability forgiveness cement and phase-change cement are proposed here for the future HTHP cementing design. Furthermore, by adapting the traditional scaling analysis approach, three zones (safe, transition and damage) with clear boundary values are showing in the (Hydro-Thermal Non-Dimensional parameter) and (Mechanical Non-Dimensional parameter) plot to give the cementing researchers and practitioners a direct and meaningful guidance about how much risk of the tensile failure that cement is taking when it is placing under the HTHP conditions. More importantly, the implications of our results are challenging the prevailing HTHP cementing principles and providing a new perspective of design and guidance by addressing the porous intrinsic nature of the cement.

1. Introduction

Properly designed and executed cementing operations are important as it is widely applied across the various earth science-related geotechnical applications, such as radioactive waste disposal, deep-well plug and abandonment (P&A), drilling and completion in unconventional reservoir, Enhanced Geothermal (EGS) reservoir, and carbon capture, utilization and storage (CCUS)(Gruber et al., 2021; Hargis et al., 2021; Koťátková et al., 2017; Olson et al., 2015; Vrålstad et al., 2019). A successful cementing job will avoid the unnecessary deficiencies during the life circle of each project which will improve the system efficiency and make the project more environment friendly. Despite the advancements in technological development of cementing materials over the last several decades, the quality of cementing is always associated with some deficiencies mainly due to the harsh environments where cement is placed (Ahmed et al., 2020; Allahvirdizadeh, 2020; Kiran et al., 2017). Taking the P&A as an example, over the years and across companies, the upper range of the reservoir’s pressure and temperature have been pushing up to 40000 psi and 600 F (DeBruijn et al., 2008; Khalifeh et al., 2020). However, the cement is originally designed for low temperature and low pressure conditions, and under such harsh conditions, its stability over an extended period of time is unknown. To leverage this problem, extensively pioneering researches have been focusing on reinforcing the cement by adding various additives, which is aiming for providing a better mechanical and hydraulic properties thus hoping for maintaining the system integrity under the extreme conditions (Cai et al., 2022; Ge et al., 2018; Katende et al., 2020; Krakowiak et al., 2018; Massion et al., 2021; Massion et al., 2022; Qin et al., 2021; Samarakoon et al., 2022).

However, the identifications of the “better” mechanical and hydraulic properties of cement under the scenario of high temperature and high pressure (HTHP) are still unclear and it does require a more comprehensive and thorough study to highlight the challenges associated with HTHP cementing so a right solution can be developed to suit these challenges. Known as the cementitious saturated porous material with permeability ranges from mili-darcy to nano-darcy (Banthia et al., 1989; Goto et al., 1981; Meng et al., 2021; Picandet et al., 2011), the cement’s behavior can be heavily influenced by the thermo-hydraulic-mechanical (THM) coupling in the porous space, especially when it is experiencing a large temperature and pressure variations. Within the permeability range mentioned above, a very lager pore pressure could be induced by the THM coupling and the pore pressure will be progressively reestablished over the time, which will also lead to the changing of the effective stress and increase the possibility of shear failure, hydraulic fracturing or even tensile failure (Ghabezloo et al., 2010). While THM coupling phenomena in the porous media has been studied by considerable researchers, their applications have been mainly focusing on the wellbore stability during drilling and fluid injection into borehole (Detournay et al., 1988; Gao et al., 2017; Song et al., 2019; Tao et al., 2010; Zhou et al., 2009). The THM coupling effect in the cementing designs and operations has rarely been properly considered and fully investigated under the HTHP conditions, which could result in jeopardizing the cementing integrity and lead to many serious consequences. More importantly, this THM coupling related issues could not be addressed by any of the current available additives, because none of them can change the intrinsic nature, i.e. the porous structure, of the cement.

To include these mutual interactions between thermal, hydraulic and mechanical systems in the non-isothermal conditions, Biot (Biot, 1977) firstly extended the tradition theory of poromechanics and include the uncoupled thermal effects by incorporating the thermos-molecular diffusion and dynamic forces using the variational Lagrangian thermodynamics approach. Later on, the thermal diffusion process was coupled in solid and fluid deformation by Derski (1979), as well as others (Bear et al., 1981; Kurashige, 1989; Smith et al., 1993). The abovementioned porothermoelastic analytical solutions are obtained by neglecting the non-linear term associated with connective heat transfer thus it is called as linear porothermoelastic model that is especially prevailing for the low permeability material (Chen et al., 2005; Delaney, 1982; Gomar et al., 2014; Wang et al., 2003). Within the framework of linear porothermoelasticity, substantial studies have been performed for coupled THM behavior of isotropic porous media to understand the mechanism of relative cases, but most of the work are assuming the fluid flux and heat flux are dominated by the pore pressure gradient and thermal gradient, respectively (Ghassemi et al., 2002; Ghassemi et al., 2009; Valov et al., 2022). That is to say, as shown in Eq.1 and Eq.2, the thermo-osmosis denoted by (fluid flex generated by thermal gradient) and mechano-caloric effects denoted by (heat flux generated by pore pressure gradient) are neglected. Noted that the mechano-caloric coefficient is also known thermal filtration coefficient (Cheng, 2016). However, as for porous material with low permeability, these two effects actually play important roles (Gonçalvès et al., 2010; Roshan et al., 2015; Trémosa et al., 2010). For example, Carnahan (1983) has shown that the thermo-osmosis flow through kaolinite can be two orders of magnitude higher than Darcy’s flow near the nuclear waste repository. Thus, when designing the cementing under the HTHP conditions, both the thermos-osmosis and mechano-caloric effects should be taken into considerations. To our best knowledge, under the HTHP conditions, their influences on the cement integrity are still unclear (is denoting for the fluid flux and is the heat flux).





By introducing so-called fully-coupled porothermoelastic model which is incorporating both of the thermo-osmosis and the mechano-caloric (thermal filtration) effects, dubbed here as “porothermoelastic-osmosis-filtration” (PTEOF), the present work used primary cementing in P&A as an example to highlight the cementing challenges that associated with HTHP conditions by considering the porous intrinsic nature of cement and its related THM coupling phenomenon. The motivations of creating the PTEOF model is to have a comprehensive understanding of the cement’s behaviors under the HTHP and to build up a general framework and solutions for future cementing studies and analysis. Drawing on the important contributions of Sarout et al. (2011) on modifying the theory of linear chemo-poroelasticity into a convenient form whereby the interpretations of the phenomenological parameters can be clarified, this paper will not only include a detailed parametric studies of PTEOF, but also it will include an important discussion of the implications of these results and provide the guidance from a new perspective of design the cementing under HTHP conditions.

2. Model Basics

Following the sign convention in Detournay et al. (1988), the positive stress is considered to be tensile within the present work. Based on the fundamental work of Cheng (2016) and Wang (2017), the constitutive equations for linear porothermoelasticity are written as follows:

 where 

whereandis volumetric strain tensor and total stress tensor, respectively. is the variation of fluid content per unit volume, is entropy density,is pore pressure change from virgin pore pressure and is temperature change from the reference temperature. The material constants include the drained bulk modulus tensor, Biot effective stress coefficient, Skempton pore pressure coefficient, coefficient of volumetric thermal expansion of porous media frame , coefficient of volumetric thermal expansion of variations in fluid content in the solid-fluid system(Cheng, 2016), andrepresents the specific heat of the porous medium at the reference temperature. From the constitutive equations, it shows that the deformation of the solid frame is caused by the effective stress and the temperature change. The fluid phase in the porous medium is not only deforming with the solid frame, but at the same time, driven by pore pressure gradient and thermal forces, causing the pore fluid entering or leaving the solid frame of unit volume. Last, the stress and temperature change will cause the change of the entropy of the porous system based on the generalized-energy relation. The entropy density is therefore becoming a function of volumetric strain of the solid frame and fluid content and the change of temperature. Thus, the constitutive equations relate and couple volumetric strain, fluid content and energy variables  with total stress, pore stress and temperature variables with the materials constant. Noted that one of these volumetric response that Eq. 3 shows can also be obtained from contraction on Eq. 4.







Based on quasi-static equilibrium (Eq.5) and the strain-displacement relations (Eq.6) and by substituting both transport laws (Eq. 1 and Eq. 2) into the mass balance equations (Eq.7 and Eq.8), the fully coupled diffusion equations (Eq. 9 and Eq. 10) could be obtained. These two diffusion equations indicate that both fluid flux and heat flux are not only dominated by the Darcy’s law and Fourier’s law, but also they are influenced by the thermal osmosis effect and thermal filtration effect. This is also where porothermoelastic-osmosis-filtration (PTEOF) so called fully coupled model roots in. One of the motivations to build up these fully coupled model is to facilitate the further studies and analysis. For example, under some circumstances where thermal osmosis or thermal filtration is not considered important, it can always take the corresponding coefficient to zero to simplify the model.









3. PTEOF solution for the primary cementing in P&A

3.1 Problem descriptions and boundary conditions

In light of the present work will showcase the PTEOF model by using the primary cementing in P&A cases where the length of the primary plug is usually 50 to 100 times larger than its diameter (Eshraghi, 2013), it is appropriate to apply the generalized plane-strain assumption where the pore pressure and thermal diffusions only appear in the isotropic plane that perpendicular to the length axis of the plug which is fully saturated. In line with the loading decomposition scheme proposed by Detournay et al. (1988) in the poroelasticity, the PTEOF model could be decomposed into three sub-loading cases to simplify the analysis, i.e. pore pressure loading (), temperature loading () and isotropic far-field stress loading (), where the superscript i is denoting by the stress field that induced by the loading mode j. Thus, the boundary conditions at the outside surface of the primary cement plug for each of the loading mode can be written as follows (the subscript f is denoting by formation, subscript c is denoting by cement). Since the PTEOF model is linear, the principle of superposition will be used as final step to obtain the final solution.

* Mode 1: , and 
* Mode 2: , and 
* Mode 3: , and 

3.2 Solution to the fully coupled diffusion equations

The solution method starts with obtaining a general solution of the fully coupled diffusion equations (Eq.9 and Eq.10). Noting that the plane-strain assumption will lead to the conclusion that fluid and thermal transport will be only directed along the radial direction in the cylindrical coordinates system. Then the first step is to eliminate  and in the diffusion equation Eq.9 and Eq.10 by substituting the constitute equations (Eq.3), this results in a new form of coupled diffusion equations could be expressed by Eq.11 and Eq.12.





Next by combining the Eq.4 to Eq.6 and taking the body force to zero, an extended form of the classical Navier equations about be obtained:



Drawing on the important contributions of Sarout et al. (2011) on modifying the theory of linear chemoporoelasticity into a convenient form whereby the interpretations of the phenomenological parameters can be clarified, we herein by using the irrotational field assumptions to simplify the process of solve the Navier equations. When displacement field is irrotational, i.e. is the gradient of a scaler, one can integrate Eq. 13 to obtain



Where  and 

Thedoes not depend on the spatial coordinates and it is a spatially uniform function that is often taking to zero for infinite or semi-infinite domain (Detournay et al., 1993). But in the case of the primary cementing in P&A, thewill not be taking as zero here. Next, by substituting Eq. 14 into Eq. 11 and Eq. 12, thecould be eliminated and it could have the coupled diffusion equations in other form as shown in Eq. 15.



Where















The diffusion equations shown in Eq.15 are coupled in terms of T and p, which can be transformed into uncoupled problems in terms of  and by using the Eigen decomposition approach developed by Sarout et al. (2011). This methodology is starting with obtaining two eigenvalues  and  of matrix. Then the Eigen-decomposition theorem (Weisstein, 2002) will allow us to define a transition matrixwhich is composed of eigenvalues and eigenvectors of , and temperature and pore pressure can be expressed as follows



Where





This transition leads directly to an uncoupled system of diffusion equations for the given by where 

Then by applying the Laplace transform to the Eigen function(Eq.18), it becomes an ordinary differential equation in terms of variable, where the is function of the coordinators in cylindrical system ρ, and the Laplace parameter s and the’s eigenvalue.

 Where 

Therefore, the original coupled diffusion equations (Eq.15) has been lead to the zeroth-order modified Bessel equation with the general solution as (Eq.19), where and  are unknowns that will be determined based on the boundary conditions. And are the zeroth-order modified Bessel functions of the first and second kind, respectively.



The symmetry conditions of the primary cementing in P&A will ensure the equals to zero, thus the pore pressure and temperature profile in Laplace domain could be expressed as follows:





Within the framework of plane-strain conditions and irrotational field, the displacement  could be obtained by integrate the Eq.14. At the same time, the strain-displacement relationship could be expressed as follows:



Where  and 

Up to now, there are totally three unknowns, i=1,2,3 in the system. The first two unknown  and are coming from the coefficient of zeroth-order modified Bessel functions of the first kind and the third unknown is coming from the spatially uniform function from Eq.14 in Laplace domain. In the process of deriving the above expressions for the pore pressure, temperature and radial total stress (all in Laplace domain), we have three equations for these three unknowns (shown in Eq.23).



Where















So that given a value of s, thecan be readily computed, thus the solution then can be numerically inverted to the time domain using Stehfest’s method (Stehfest, 1970), which has been proved to be efficient in poroelastic problems and its details are shown in Appendix B.

4. Numerical Analysis and Discussions

The notations are used for the modeling are summarized in Table 1 at Appendix A. The temperature difference between the formation and cement is assigned as; the virgin pore pressure of the formation is setup as Pa and far-field isotropic stress is setup as Pa (Snee et al., 2018; Xu et al., 2015; Zoback et al., 2003). In this section, we will firstly discuss the pore pressure profile and temperature profile that induced by three different loads, respectively. Lastly, the superposition results of the effective stress will be summarized.



4.1 Pore pressure responses and thermal osmosis effect

The evolution of pore pressure in response to different loadings is a key component in poromechanical analysis. It is therefore of interest to firstly plot the evolution of pore pressure under three different loading modes given the boundary conditions that defined in the last section, and the influence of the factors of interest on the pore pressure will be analysis and discussed later.

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Figure 1a. Pore pressure change due to mode 1 loading, where the curves grade from gray to black as time increases

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Figure 1b. Pore pressure change due to mode 2 loading, where the curves grade from gray to black as time increases

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Figure 1c. Pore pressure change due to mode 3 loading, where the curves grade from gray to black as time increases

Fig.1a and 1b illustrate the evolution of pore pressure profile along the radius in response to mode 1 loading (pore pressure loading) and mode 2 loading (temperature loading). Under the mode 1 loading, it is similar to the classical diffusion process, the pore pressure near the surface (r=1) is instantly raised to the level of pore pressure loading and then the pore pressure is gradually diffusing towards in the center (r=0). However, in contrast of gradual and smooth diffusion process, the pore pressure induced by mode 2 firstly peaks near the surface area. At the surface, the pore pressure returns to zero due to the boundary conditions setup. The pore pressure, however, decreases toward the inner core (because of the slow diffusion progress due to low hydraulic conductivity) hence a peak profile is developed. As time progresses, the pore pressure peak declines due to fluid diffusion, and at the same time, the inner core becomes heated as well and so the peak is moving toward to the center. At even larger time, when the entire specimen is heated but the pore pressure is not yet dissipated due to the small fluid diffusivity. As time further increases, the entire pore pressure dissipates and return to zero everywhere. For the pore pressure that induced by the mode 3 (isotropic far-field stress), as shown in Fig.1c, it is firstly arrives the highest value right after the loading (again, pore pressure at surface reduces to zero due to the boundary conditions setup), but it gradually declines to zero due to the dissipation process as the time process. It is noticeable that the magnitude of the pore pressure induced by mode 3 is much smaller compared to the pore pressure induced by mode 1 and mode 2. Even at their highest level which is right later the loading is applied, the ratio of mode 3 induced pore pressure over the mode 1 and mode 2 induced pore pressure is around 0.01%-0.02%. This is also in line with the findings from the classical pure poroelastic model that Detournay et al. (1988) concluded.

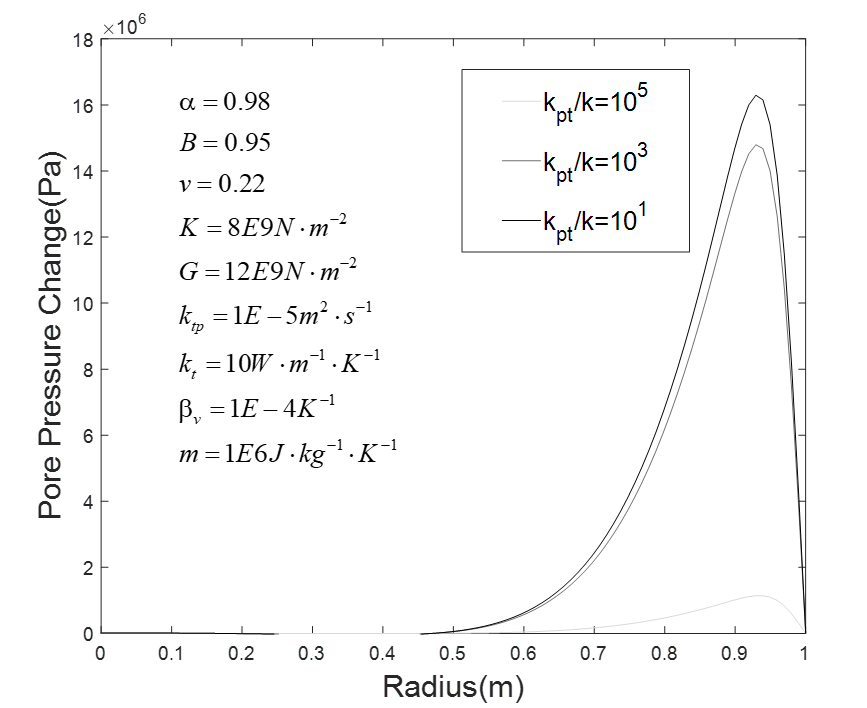


Figure 2. Various ratio of (thermo-osmosis’s influence) on the induced pore pressure (days)

Since the peak pore pressure usually occurs at the early-time, we will use the smaller time at days after loading to illustrate the thermal osmosis effect on the induced pore pressure profile. Three different ratios (1, 103, 105) of are selected to show the different developments of the temperature induced pore pressure (Figure 2). Recalling the definition of the thermal-osmosis coefficient which can be understood as the passage of a fluid that is driven by a temperature gradient but against the hydrostatic pressure (Denbigh, 1949). Taking the early time as example, all three cases are showing a peak pore pressure near the surface region. However, the magnitude of the peak induced pore pressure is reduced when the thermos-osmotic coefficient is significantly larger than the hydraulic conductivity (i.e., =105).

4.2 Temperature response and thermal filtration effect

The temperature profiles under the mode 2 loading (temperature loading) is shown in Figure 3. As the time proceeds, temperature is increasing monotonically from surface towards the inner core of the plug and finally arrives the equilibrium which is equilibrium to the boundary conditions at larger time. Three different ratios of thermal conductivity over the thermal filtration coefficients are selected to show the impact of thermal filtration effect on the temperature profile that is changed by the pore pressure gradient. As shown in Figure 4, among the three selected ratios, the maximum temperature difference induced by the pore pressure gradient is within 1 degree Kalvin. Furthermore, when the thermal conductivity is significantly larger than the thermal filtration coefficient (=105), the induced temperature difference are negligible under the current model settings and input.

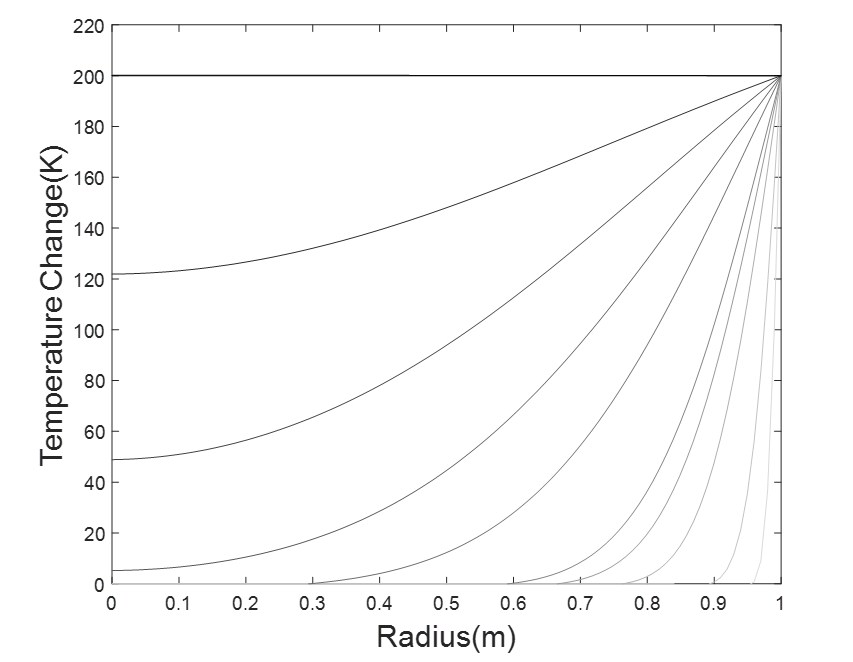


Figure 3. Temperature profile development under the loading mode 2, where the curves grade from gray to black as time increases

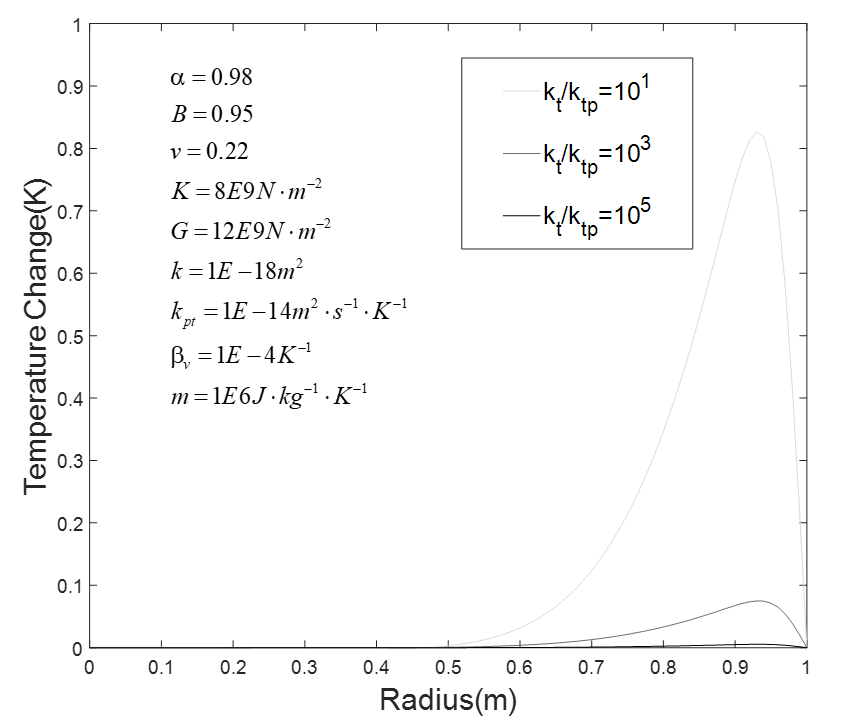


Figure 4. Various ratio of (thermal filtration’s influence) on the temperature profile change

4.3 Effective Stress

Many pioneering researches have shown that the effective stress is a key factor to control the behaviors of the porous medias in the geotechnical and earth applications (Khalili et al., 2004; Skempton, 1984). Specifically, the increase pore pressure will reduce of effective stress which can increase the possibility of many types of failures. Thus, appropriate estimation and calculation of total effective stress will greatly enhance the understanding of the behaviors of the porous system. As shown in figure 5 where the total effective radial stress (after superposition from three loading modes) is plotted (again, tensile is positive). After the cement is placed under the boundary conditions that mentioned above, at the early stage, the induced pore pressure will reduce the radial effective stress near the surface region, but the entire sample is still under the compressive condition. However, as the diffusion process is very slow inside the sample but the pore pressure is kept cumulating due to the HTHP conditions, the effective stress will be further reduced and the tensile region will be created inside the core. At the later stage, when induced pore pressure is fully dissipated, the radial effective stress will return to compressive again. This demonstration of the whole diffusion process of PTEOF model with given input indicates that cementing under the high temperature and high pressure conditions will definitely induce a high value of pore pressure, which will lower the effective stress and even will generate the tensile region as the results of pore pressure slowly diffuse inward. These will increase the possibility of inducing different kinds of failures to the cement and jeopardizing the integrity of the cementing system which will cause more severe sequences.



Figure 5. Total effective stress of cement under the conditions of HTHP, where the curves grade from gray to black as time increases (boundary conditions and input values of each parameter are stated at the beginning of this section)

4.4 Fully parametric studies

It is worth to mention that the system behaviors discussed in the last section are depending on the different values of input parameters and the mutual interaction and group effect among them. If some of these parameters are taking different values, the results can be much different. Thus, these phenomena will be investigated and discussed in this section. The analysis is starting with assigning values (within a certain range that summarized in Table 2 in Appendix A) to each parameter by the Monte Carlo method which is class of techniques for randomly sampling (Shapiro, 2003). Then these variables with given values will be substituted into the constitutive equations and loading decomposition procedures that discussed previously to obtain the effective radial stress, which will be categorized into tensile and compressive groups. The details of the above mentioned procedures are summarized in flowchart in Figure 6.

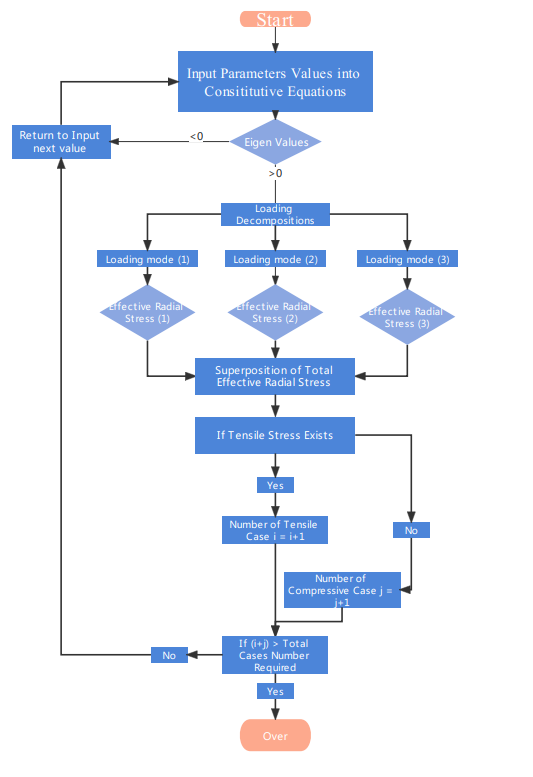


Figure 6. The Algorithm that is used to construct the fully parametric studies

4.4.1 Pairwise Bivariate Analysis for Each Variable

Following the procedures that flowchart described in Figure 6, total of 3000 cases are calculated and then categorized into two groups – tensile (1727 cases) and compressive (1213 cases). Next, the relationship between each variable is firstly virtualized by the pairwise bivariate distributions that is shown in Appedix C. The off-diagonal elements are scatter plots which display the correlation among two variables and give a valuable insight suggesting the distribution features of these variables. The lower and upper triangles are symmetrical. The diagonal elements are univariate distribution plots which are drawn to show the probability density function of each variable. It can be found that in the events of tensile case, the specific heat and permeability are more concentrated at their lower range and the thermal conductivity and the thermal expansion coefficient difference are more concentrated at their higher range, whereas the rest of the parameters are almost evenly distributed at their whole range. This indicates a general trend that, within the framework of PTEOF, a higher values of  and lower value of and will increase the probability of generating the tensile radial effective stress.

A very interesting point can be found and discussed here. The current cement designing principles are that the lower hydraulic conductivity, the better system behavior. This cement designing philosophy is effective under most of working conditions (ambient temperature and pressure) that cement is originally built for. However, when the cementing is working under the HTHP conditions and taking the poromechanics into considerations, our results show that the lower permeability will actually increase the probability of the tensile case which is considered as detrimental to the integrity of the cementing system. This is mainly due to that under the framework of porous media, the lower permeability will largely slow down the pore fluid diffusion process when the pore pressure is rapidly built up within the cement due to the HTHP boundary conditions. The accumulated pore fluid within the clogged porous system will build up excess pore pressure and the effective stress will be thus reduced or even extend into the tensile region which will eventually jeopardize the whole system. That is to say, blindly pursing the low permeability regardless the intrinsic porous properties of the cement itself will instead victimize cement’s functions that are providing mechanical support and zonal isolations. This phenomenon is firstly named here as permeability penalty. To avoid the permeability penalty, perhaps the most favorable solution for cementing design under HTHP is to main the permeability at certain ranges which can achieve the sealing function, but at the same time, it will allow the diffusion of pore pressure that is built up by the HTHP conditions and not cause the unnecessary damage that induced by the excess pore pressure. This permeability-forgiveness design is actually very popular in the pavement design of permeable porous system (PPS) (Scholz et al., 2007) where one or two special drainage layer with relative higher permeability are designing for a more effective and efficient way to reduce the runoff rates in the storm while providing a hard surface for the traffic flow. The PPS design has been successfully turned into wide variety off residential, commercial and industrial applications in the last two decades (Drake et al., 2013).

It also worth mentions that the specific heat capacity is rarely get any attention in the previous research in HTHP cementing design. However, based on our results, it does play a very important role which is similar to the role of thermal conductivity as a large specific heat capacity will slow down the heat transfer process and the thermal induced pore pressure build up progress will be delayed and the pore fluid will have more time to dissipate. This provides a totally new perspective of creating a phase-change cement with high volume specific heat capacity but low thermal conductivity that is specially designed for HTHP conditions, which is having very promising and wide applications.

4.4.2 New Proposed Dimensionless Parameters

Although the pairwise bivariate distributions shown in Appendix C gives a full picture of how each parameter will individually influence the system behaviors, the mutual interactions of different properties and their group effect can not be analyzed from it. To solve this problem, scaling analysis is one the classical methods. It is usually used to reducing complex physical problem to a simpler version prior to obtaining a quantitative answer and can grasp the effects of various physical phenomena at the same time (Abbas et al., 2013). We indeed can follow the typical procedures to conduct the scaling analysis to investigate the mutual interaction and group effect of the PTOEF model. However, the new dimensionless numbers that is resulting from the scaling process is often in a very complicated expressions and lacking of a direct meaning at the physical level, which can be very ambiguous in terms of many engineering applications. Thus, inspired by the spirit of the methodology of scaling analysis, we here adapted to manually compose two new composite dimensionless parameters with clearer physical meaning and then construct a new plot by using these two parameters that help us to distinguish different system behaviors that can account for the mutual interactions and group effects among these different parameters. It is therefore easier for many cementing practitioners to pick them up and apply them to the real engineering problem. Thus, two new parameters are hereby proposed as follows:

The first dimensionless numberconsists of Biot coefficient, Skempton coefficient, thermal expansion coefficient of fluid and solid, thermal diffusivity and fluid diffusivity, where the last two terms are obtained from the diagonal elements from the coefficient matrix in Eq. 15. This new proposedmainly contains the hydro-thermal properties of the porous media. It is thus named as Hydro-Thermal Non-Dimensional parameter (HTND). The second proposed parameter is consisting of three mechanical related components – Shear Modulus, Bulk Modulus and Poisson’s ratio. Thus, it is named as Mechanical Non-Dimensional parameter (MND).

Given the new proposed parameters, a new plot can by generated to show the tensile and compressive cases spatial distributions based on( HTND) and(MND). In Figure 8 (a), the same 3000 cases are replotted where the red dot is representing the tensile case and the green dot is representing the compressive case. It can be clearly seen that the spatial distributions of the tensile and compressive caese in the- plot are heavily depending on thewhereas both tensile and compressive cases are almost evenly distributed along theaxis. This is indicating that the influcne of the on the system is dominating when compraring with the. Futhermore, in Figure 8 (b) where theis equally individed into 300 intervals and the probablity of generating the tensile cases in each interval is calcalated. It is shown that in the area where theare smaller than 0.07 are fully occupie by the compressive cases and there is no single tensile case exists in this region. It is thus called the safe zone which means no tensile failure would occur within this range under the framework of the PTEOF model. When the is increasing from 0.07 to 1, the probablity of generating the tensile cases are gradually increaing up to 100%. Both tensile and compressive cases could happen in this region, but with a higher value of , the possibility of generating the tensile case will be higher. This ara is thus called as transition zone. When the is above 1, no compressive case is existing within this range and it is fully occupied by the tensile cases which means the probality of generting the tensile cases is 100%. It is thus named as damage zone which means when designing the cement under the HTHP conditions, if the resulting is landing above 1, it certainly will have the tensile damage occurredin the cement.

Drawing the conclusions above, it can be easily seen that the Bulk Modulus, Shear Modulus and Poisson’s ratio, which are considered as three of the most important mechanical properties, are having very limited contribution to the system behavior when the is presenting. This is actually challenging the current HTHP cementing design principles that are always prioritizing to making the cement stronger and more ductile, which is hope for the cementing system will thus have more resilience and has less possibility to failure. However, based on our results, it actually may not help to maintain the cement integrity under the HTHP conditions. That is to say, when designing the cement under the HTHP conditions, rather than giving the highest priority to these mechanical properties, more attentions should be put into the components that included in the which should keep its value below 0.07 to avoid any tensile damage.

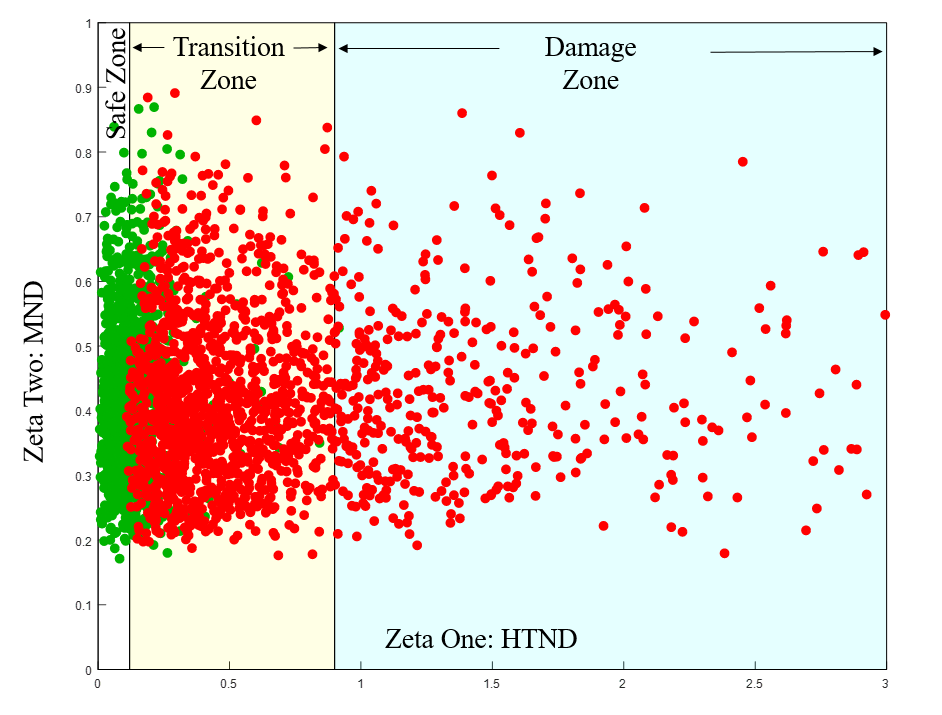


Figure 8 (a) (HTND) and(MND) plot with tensile cases (red color) and compressive cases (green color).

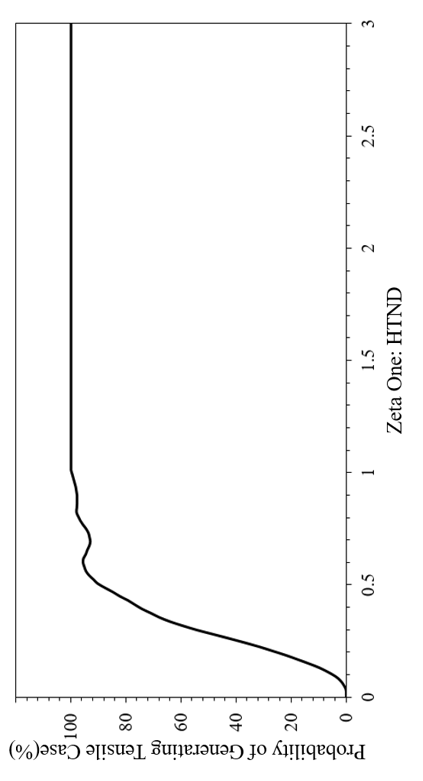


Figure 8 (b) The probability plot of generating tensile cases on different  (HTND) values

5. Conclusions

Using the primary cementing in P&A as example, the fully coupled porothermoelastic-osmosis-filtration (PTEOF) model is presented here. It has been demonstrated that under HTHP conditions, the induced pore pressure in the low permeability material such as cement can be very large to greatly reduce the effective stress and can even extend it to the tensile region. This reduced effective stress phenomenon will in general increase the probability to cement failure that may occur in different types. By the pairwise bivariate analysis, the properties that are important for the HTHP cementing are parsed out. Based on these results, two new concepts of permeability forgiveness cement and phase-change cement are proposed here for the future HTHP cementing design. Furthermore, by adapting the scaling analysis approach, three zones (safe, transition and damage) with clear boundary values are showing in the (HTDN) and (MDT) plot to give the cementing researchers and practitioners a direct and meaningful guidance about how much risk of the tensile failure that cement is facing when it is placing under the HTHP conditions.

More importantly, the implications of our results are challenging the prevailing HTHP cementing principles and providing a new perspective of design and guidance by addressing the porous intrinsic nature of the cement. The current cementing improvement practices, which are adding different additives into cement to enhance its mechanical properties, will not change the fact that the new additive mix is still a porous media. The new additive can change the pore space into various sizes even into nano-scale though. However, without considering the underlying physical principles governing the mechanical behaviors under the framework of porous media, it sometimes will jeopardize the whole system instead. Our results indicate that when cementing under HTHP conditions, the mechanical properties such as shear modulus and Poisson’s ratio may not be the most important parameters to maintain the integrity of cement and the low permeability will actually cause the tensile stress to damage the system. Thus, we are proposing a new cementing design principle of permeability forgiveness which is allowing for some tolerance of permeability to avoid the permeability penalty.

Another important implication from our results is that instead focusing on enhancing the mechanical properties, thermal properties such as thermal conductivity, specific heat capacity, thermal expansion coefficient ratio and the thermo-osmosis coefficient also deserves more attention. Our analysis shows that the magnitude of thermal induced pore pressure is resulting from the competing process of thermal diffusion and fluid diffusion. Thus, slowing down the thermal transfer process becomes a key step to prevent the thermal load induced pore pressure damage. Certainly, reducing the thermal conductivity is the most direct method to achieve this goal. However, our results show that increasing the specific heat capacity is another efficient way to slow down the pore pressure build up progress. More important, it provides a totally new perspective of creating a phase-change cement with high volume specific heat capacity but low thermal conductivity that is specially designed for HTHP conditions, which is having very promising and wide applications.

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Appendix A: Table 1. Notations

|  |  |  |
| --- | --- | --- |
| Symbol | Definition | Unit |
|  | Biot effective stress coefficient | --- |
|  | Drained thermoelastic effective stress coefficient | N⋅m-2⋅K-1 |
|  | Skempton pore pressure coefficient | --- |
|  | Coefficient of volumetric thermal expansion of solid | K-1 |
|  | Coefficient of volumetric thermal expansion of porosity | K-1 |
|  | Coefficient of volumetric thermal expansion of fluid | K-1 |
|  | Coefficient of volumetric thermal expansion for variation in fluid content | K-1 |
|  | Drained coefficient of volumetric thermal expansion of porous medium frame | K-1 |
|  | Specific heat of the porous medium at reference temperature | J⋅kg−1⋅K−1 |
|  | Fluid flux | m⋅s−1 |
|  | Heat flux | J⋅m−2⋅s−1 |
|  | Permeability | m2 |
|  | Thermal conductivity | W⋅K−1⋅m−1 |
|  | Mechano-caloric coefficient (Thermal filtration) | m2⋅s−1 |
|  | Thermo-osmosis coefficient | m2⋅s−1⋅K−1 |
|  | Poisson’s ratio | --- |
|  | Drained bulk modulus | N⋅m-2 |
|  | Shear modulus | N⋅m-2 |
|  | Thermal Diffusivity | m2/s |
|  | Fluid Diffusivity | m2/s |

Table 2. Lower boundary and upper boundary for Monte Carlo Sampling (the unit of each parameter is same as the unit defined in Table 1)

|  |  |  |
| --- | --- | --- |
|  | Lower Boundary | Upper Boundary |
|  | 0.55 | 0.95 |
|  | 0.58 | 0.98 |
|  | 1E-5 | 1E-4 |
|  | 8E9 | 16E9 |
|  | 12E9 | 24E9 |
|  | 0.22 | 0.32 |
|  | 1E-17 | 1E-16 |
|  | 1E1 | 1E2 |
|  | 1E-5 | 1E-4 |
|  | 1E-14 | 1E-13 |
|  | 1E6 | 1E7 |

Appendix B: Details of Stehfest’s method

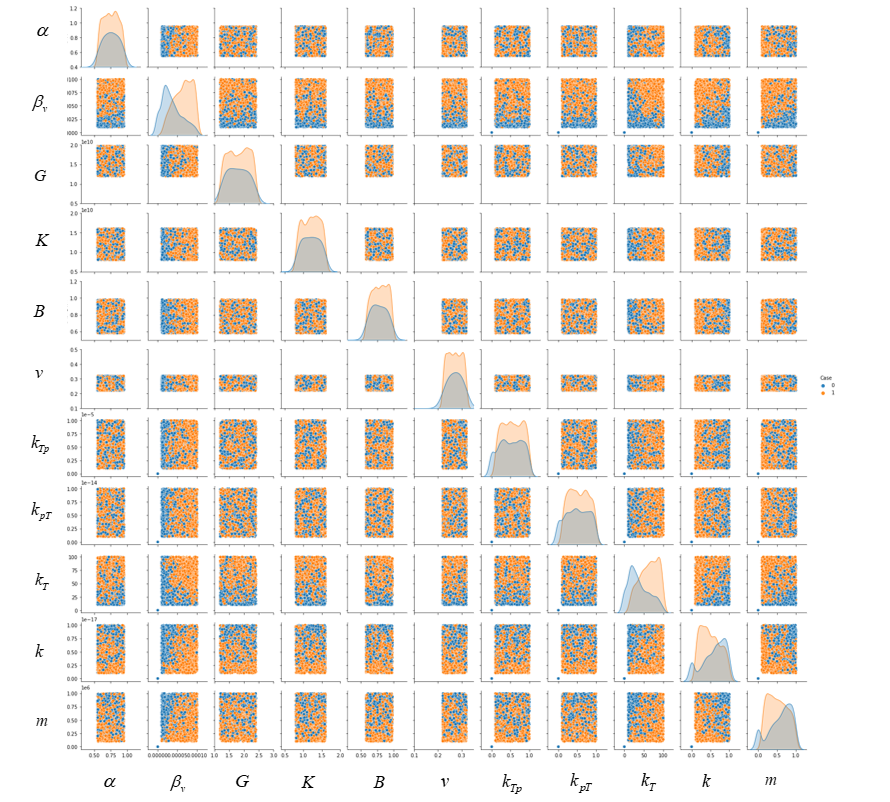
For a given function p with Laplace transform, Stehfest’s method can be expressed as, taking pore pressure for example:



Where the coefficient are given by



Appendix C:



Pairwise bivariate distributions for eleven variables (each orange color dot represents tensile case and blue color dot represents compressive case)

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